[11-01-10-T12]

Local extreme values of f

First derivative

If $f'(x) > 0$ for all x in (a, b), then f is an increasing function on $[a, b]$.

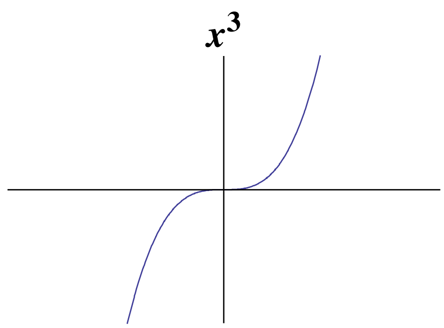
If $f'(x) < 0$ for all x in (a, b), then f is a decreasing function on $[a, b]$.

Given this, the following should seem reasonable.

Suppose f is continuous on $[a, b]$ and differentiable on (a, b) and $c \in (a, b)$.

If $f'(x)$ changes sign at c, then $f(c)$ is a local extreme value of f.

Do not get too excited if $f'(x) = 0$ at c, because $f(c)$ may or may not be an extreme value of f on (a, b). For example,



■ Second derivative

If $f''(x) > 0$ for all x in (a, b), then f is concave up on (a, b).

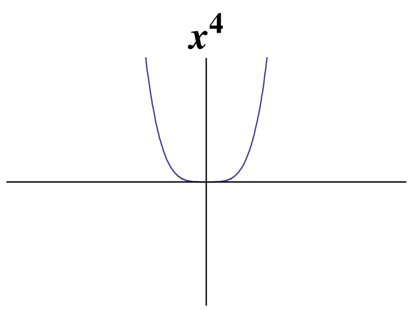
If $f''(x) < 0$ for all x in (a, b), then f is concave down on (a, b).

Given this, the following should seem reasonable.

Suppose f is twice differentiable on (a, b) and $c \in (a, b)$.

If $f''(x)$ changes sign at c, then $(c, f(c))$ is an inflection point of the graph of f.

Do not get too excited if $f''(x) = 0$ at c, because $f(c)$ may or may not be an inflection point. For example,



If you keep the above ideas in mind and think about the function you are given, finding extreme values and producing a graph of the function should not be too hard.

■ Graphing functions (a checklist)

- **A. Domain**
- **B. Intercepts**
- **C. Symmetry**
- **D. Asymptotes**
- **E. Intervals of increase and decrease**
- **F. Local maximum and minimum values**
- **G. Concavity and points of inflection**

■ Example 1

Discuss and graph the function $f(x) = \frac{1}{x^2-1}$.

■ A. Domain

$\text{Dmn}_f = \mathbb{R} - \{-1, 1\}$. We know we will be interested in the behavior of f when x is near -1 and 1.

■ B. Intercepts

When $x = 0$, $f(x) = -1$.

■ C. Symmetry

f is an even function, because $f(x) = f(-x)$. The graph will be symmetric with respect to the y-axis, so an analysis of f for $x > 0$ will provide all we need to graph f on its entire domain.

■ D. Asymptotes

Since f is undefined at $x = -1$, $x = 1$. We note that:
$x \to -1^-$, $y \to \infty$;
$x \to -1^+$, $y \to -\infty$;
$x \to 1^-$, $y \to -\infty$;
$x \to -1^+$, $y \to \infty$.

■ E. Intervals of increase and decrease

$\frac{dy}{dx} = \frac{-2x}{(x^2-1)^2}$. Since the denominator is positive, the sign of f' is determined by the numerator.

	(∞, −1)	**(−1, 0]**	**[0, 1)**	**(1, ∞)**
f ' (x)	+	+	−	−
f (x)	INC	INC	DEC	DEC

■ F. Local maximum and minimum values

Since f is increasing left of zero and decreasing right of zero, $(0, -1)$ is a local minimum.

■ G. Concavity and points of inflection

$\frac{d^2 y}{dx^2} = \frac{2(3x^2+1)}{(x^2-1)^3}$. Since the numerator is positive, the sign of f'' is determined by the denominator. The numbers -1 and 1 are split points.

	(∞, −1)	**(−1, 1)**	**(1, ∞)**
f '' (x)	+	−	+
f (x)	UP	DWN	UP

The curve has no point of inflection, since -1 and 1 are not in the domain of f.

Graph

The following graph is consistent with our analysis of *f*.